ATAR course examination, 2017

Question/Answer booklet

MATHEMATICS METHODS

**Section Two:
Calculator-assumed**

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student number: In figures

In words _____

_____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	52	35
Section Two: Calculator-assumed	11	11	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (99 Marks)

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 10 (3 marks)

Use the quotient rule to show that $\dfrac{d}{dx}\tan(x) = \dfrac{1}{\cos^2(x)}$.

Question 11 **(9 marks)**

A pizza shop estimates that the time X hours to deliver a pizza from when it is ordered is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{4}{3} - \dfrac{2}{3}x , & 0 < x < 1 \\ 0 , & \text{otherwise.} \end{cases}$$

(a)   What is the probability of a pizza being delivered within half an hour of being ordered?

(2 marks)

(b)   Calculate the mean delivery time to the nearest minute. (3 marks)

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(c)   Calculate the standard deviation of the delivery time to the nearest minute.   (4 marks)

Question 12 **(13 marks)**

The Slate Tablet Company produces a variety of electronic tablets. It wants to gather information on consumers' interest in its tablets.

(a) In each of the following cases, comment, giving reasons, whether or not the proposed sampling method introduces bias.

(i) A Slate Tablet Company representative stood outside an electronics store on a Saturday morning and asked people entering the store 'If you were to purchase an electronic tablet would you choose a Slate Tablet or an inferior brand?' (2 marks)

(ii) Fifteen hundred randomly selected mobile phone numbers were telephoned and people were asked 'Which brand of electronic tablet do you prefer?' (2 marks)

A common problem with a particular tablet is screen failure. The manufacturer of Slate Tablets has found that 1% of its tablet screens will fail within three years. A sample of 200 tablets is taken. Let the random variable X denote the number of tablets that have screen failure within three years in the sample of 200.

(b) What is the distribution of X? (2 marks)

(c) What is the probability that more than four tablets will have screen failure within three years? (2 marks)

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In a random sample of 200 Slate Tablets, four of them had screen failure within three years.

(d)   Calculate an approximate 95% confidence interval for the proportion of tablets that have screen failure within three years. Give your answer to four decimal places.   (3 marks)

(e)   The company's quality control department wants the proportion of tablets with faulty screens to be between 0.5% and 1%. Based on your confidence interval, decide whether the quality control department is meeting its target. Justify your decision.   (2 marks)

Question 13 **(9 marks)**

Ravi runs a dice game in which a player throws two standard six-sided dice and the sum of the uppermost faces is calculated. If the sum is less than five, the player wins $20. If the sum is greater than eight, the player wins $10. Otherwise the player receives no money.

(a) Complete the table below. (2 marks)

Amount won			
Probability			

(b) What is the expected amount of money won by a player each time they play? (2 marks)

(c) Liu Yang decides to play the game. If Ravi charges her $5 to roll two dice, who is likely to be better off in the long-term? Explain. (3 marks)

(d) If Ravi wants to make a long-term profit per game of 20% of what he charges, what should he charge a player to roll the two dice? (2 marks)
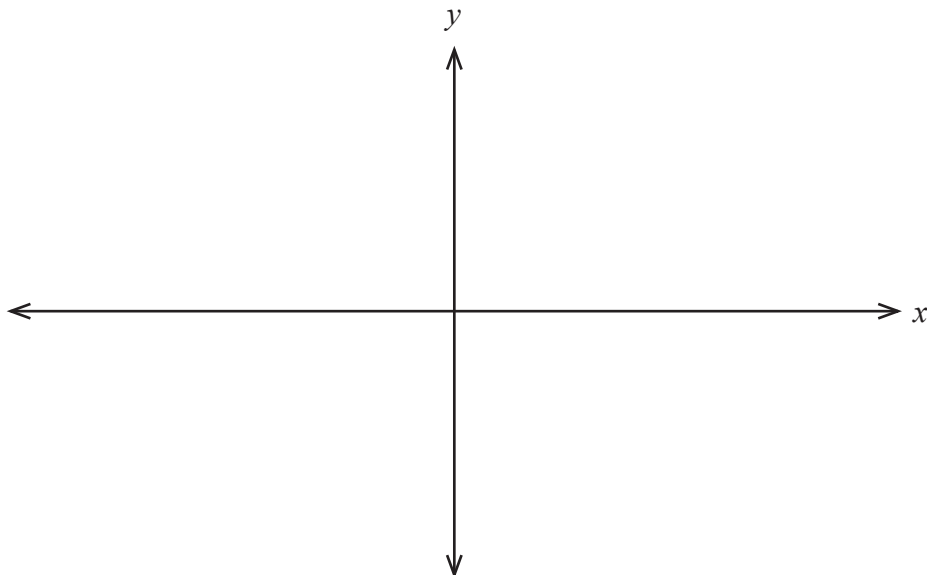
Question 14 **(9 marks)**

Let $f(x) = x \ln(x + 3)$.

(a) Use calculus to locate and classify all the stationary points of $f(x)$ and find any points of inflection. (5 marks)

(b) On the axes provided sketch the graph of $f(x)$, labelling all key features. (4 marks)

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Question 15 **(10 marks)**

The volume $V(h)$ in cubic metres of a liquid in a large vessel depends on the height h (metres) of the liquid in the vessel and is given by

$$V(h) = \int_0^h e^{\left(-\frac{x^2}{100}\right)} dx, \ 0 \le h \le 15.$$

(a) Determine $\dfrac{dV}{dh}$ when the height is 0.5 m. (2 marks)

(b) What is the meaning of your answer to part (a)? (1 mark)

(c) The height of the liquid depends on time t (seconds) as follows:

$$h(t) = 3t^2 - t + 4, \ t \ge 0.$$

(i) Determine $\dfrac{dh}{dt}$ when the height is 6 m. (2 marks)

(ii) Use the chain rule to determine $\dfrac{dV}{dt}$ when the height is 6 m. (2 marks)

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(iii) Given the volume of the liquid at 2 seconds is 8.439 m^3, use the incremental formula to estimate the volume 0.1 second later. (3 marks)

Question 16   **(8 marks)**

A group of biologists has decided that colonies of a native Australian animal are in danger if their populations are less than 1000. One such colony had a population of 2300 at the start of 2011. The population was growing continuously such that $P = P_0 e^{0.065t}$ where P is the number of animals in the colony t years after the start of 2011.

(a)   Determine, to the nearest 10 animals, the population of the colony at the start of 2014.

            (2 marks)

(b)   Determine the rate of change of the colony's population when $t = 2.5$ years.   (2 marks)

(c)   At the beginning of 2017, a disease caused the colony's population to decrease continuously at the rate of 8.25% of the population per year. If this rate continues, when will the colony become 'in danger'? Give your answer to the nearest month.   (4 marks)

Question 17 **(6 marks)**

A beverage company has decided to release a new product. 'Joosilicious' is to be sold in 375 mL cans that are perfectly cylindrical. {Hint: $1\ mL = 1\ cm^3$}

(a) If the cans have a base radius of x cm show that the surface area of the can, S, is given

by: $S = 2\pi x^2 + \dfrac{750}{x}$. (2 marks)

(b) Using calculus methods, and showing full reasoning and justification, find the dimensions of the can that will minimise its surface area. (4 marks)

Question 18 **(11 marks)**

Alex is a beekeeper and has noticed that some of the bees are very sleepy. She takes a random sample of 320 bees and finds that 15 of them are indeed so-called *lullabees* that fall asleep easily.

(a) Calculate the sample proportion of lullabees. (1 mark)

(b) Determine a 90% confidence interval for the true proportion of lullabees, rounded to four decimal places. (3 marks)

(c) What is the margin of error in the above estimate? (2 marks)

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It turns out that the true proportion of lullabees is 0.02.

(d)   Now that Alex knows this, she decides to take a new sample.

   (i)   Suppose a new sample of 290 bees was taken. Given that the true proportion
           of lullabees is 0.02, what is the probability that the sample proportion in this new
           sample is at most 0.03?   (3 marks)

   (ii)   If Alex takes a larger sample, will the above probability increase or decrease?
           Explain.   (2 marks)

Question 19 **(12 marks)**

A global financial institution transfers a large aggregate data file every evening from offices around the world to its Hong Kong head office. Once the file is received it must be processed in the company's data warehouse. The time T required to process a file is normally distributed with a mean of 90 minutes and a standard deviation of 15 minutes.

(a) An evening is selected at random. What is the probability that it takes more than two hours to process the file? (2 marks)

(b) What is the probability that the process takes more than two hours on two out of five days in a week? (3 marks)

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The company is considering outsourcing the processing of the files.

(c) (i) A quotation for this job from an IT company is given in the table below. Complete the table. (1 mark)

Job duration (minutes)	$T \leq 60$	$60 < T < 120$	$T \geq 120$
Probability			
Cost Y ($)	200	600	1200

(ii) What is the mean cost? (2 marks)

(iii) Calculate the standard deviation of the cost. (2 marks)

(iv) In the following year, the cost (currently Y) will increase due to inflation and also the introduction of an additional fixed cost, so the new cost N is given by: $N = aY + b$. In terms of a and/or b, state the mean cost in the following year and the standard deviation of the cost in the following year. (2 marks)

Question 20 **(9 marks)**

A model train travels on a straight track such that its acceleration after t seconds is given by $a(t) = pt - 13$ cm/s², $0 \le t \le 10$, where p is a constant.

(a) Determine the initial acceleration of the model train. (1 mark)

The model train has an initial velocity of 5 cm/s. After 2 seconds it has a displacement of –50 cm. A further 4 seconds later its displacement is 178 cm.

(b) Determine the value of the constant p. (4 marks)

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(c)   When is the model train at rest?   (2 marks)

(d)   How far has the model train travelled when its acceleration is 47 cm/s^2?   (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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